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Mathematical Formulation of Free Piston Stirling Cooler for Domestic Refrigeration using Loss Factor

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Abstract— Free Piston Stirling Cooler (FPSC) is a novel innovation in refrigeration field, which works on reversed Stirling cycle. A simplified model for numerical analysis has been developed and results are displayed graphically and in tabular format. The entire refrigeration system was evaluated theoretically and graphically for the performance vis-à-vis coefficient of performance, heat absorbed and work input required of the FPSC unit.

Index Terms— Free Piston Stirling Cooler, FPSC, Numerical Analysis of FPSC, Stirling Coolers, Stirling Refrigeration,Loss factor

1 INTRODUCTION

Refrigeration refers to the process of heat removal from an enclosed space in order to lower the temperature in that region. The process has its uses in commercial as well as domestic applications and thus it has created a large impact on industry, lifestyle and agriculture sector.

While the commercialized variable capacity compressor technology or the linear compressor technology which currently seems to be at the development stage can be possible successors of the conventional vapor compressors in the future; research on alternative refrigeration methods such as Stirling cycle, magnetic cooling and thermoacoustics have reached to a certain level and these technologies can also be considered as challenging alternatives to conventional compressors.

A free piston Stirling cooler is basically a closed system refrigerator which uses eco-friendly refrigerants such as air, hydrogen and helium. Free Piston Stirling Cooler is abbreviated as FPSC and is a pressure vessel which operates by shuttling a certain amount of Helium gas back and forth by the combined movement of the piston and the displacer and can be determined by a cold head where thermal energy is extracted from the surroundings and a warm head where heat is rejected to the environment.

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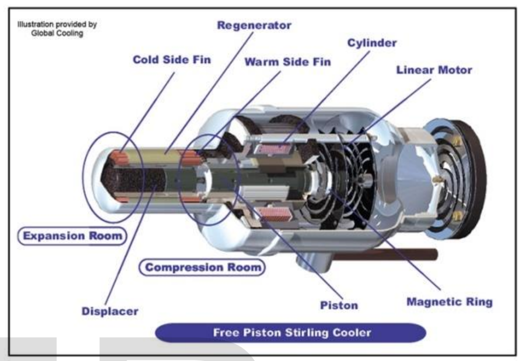

Fig. 1 Free Piston Stirling cooler (cut-section) [i]

Past calorimetric studies on free piston Stirling coolers showed that a COP value higher than 2.5 could be reached at certain operating conditions depending on these cold and warm head temperatures [1, 2]. Another advantage of the free piston Stirling coolers is declared to be the high COP levels at low heat loads even lower than 30 W – which can be maintained by modulating the input voltage and hence the refrigeration capacity of the Stirling cooler. Although there are several heats transfer mechanisms – such as forced fluid convection, forced air convection or thermosyphon method – that can be applied while integrating the Stirling cooler to a domestic refrigerator cabinet, recent studies has focused on the thermosiphon system since little additional power is needed to circulate the heat transfer media.

In this paper, we shall focus on direct cooling, in which the cold head of the FPSC unit is directly in contact with the medium (air) in the refrigeration space. Free or natural convection occurs in this case, although forced convection can easily be achieved by installing a fan with suitable rating near the cold heat. In this way the heat transfer coefficient may be controlled as desired.

International Journal of Scientific & Engineering Research, Volume 7, Issue 11, November-2016
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2 Working Principle of FPSC

2.1 Process Diagram

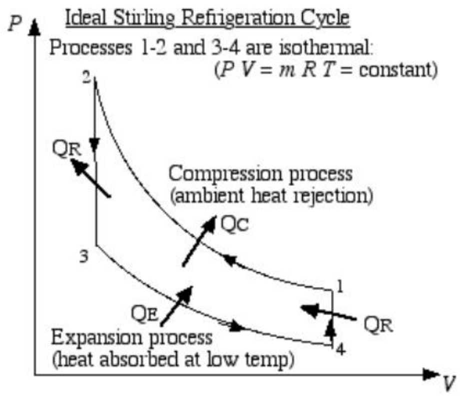

Fig. 2 Ideal reversed Stirling cycle [ii]

Process 1-2:- Isothermal Compression Piston moves towards regenerator,displacer remains stationary. Heat transfer from the working fluid to external sink at ambient temperature, T_c.

Process 2-3:- Constant volume heat release; the piston and displacer moves simultaneously keeping the volume between them constant. Heat transfer from working fluid to regenerator matrix. The fluid is transferred through the regenerator to expansion space during which the fluid is cooled.

Process 3-4:- Isothermal expansion; Piston being stationary the displacer continuously moves away from the regenerator. As expansion proceeds the pressure decreases. Heat transfer takes place to working fluid from external source at refrigerating temperature, T_E.

Process 4-1:- Constant volume heat absorption; Finally piston and displacer move simultaneously to transfer the working fluid back through the regenerative matrix from the expansion space to compression space. Thus transferring heat back from the matrix to the working fluid.

2.2 Actual Working

Ideally Stirling cycle consists of two isothermal and two constant volume regenerative processes. The more realistic model should account for the following factors.

a) ***Mass Distribution***: - The Stirling cycle assumes that the whole of the working fluid at the four salient points of the cycle is concentrated either in compression space or expansion space. This will require the regenerative matrix to have zero void volume. Since fluid has to pass through the heat exchanger, it must have some flow passage and hence a finite void volume. The void volume will reduce compression ratio with the corresponding effect on pressure compression ratio and thus also a modification in P-V diagram for the cycle

b) ***Harmonic Piston Motion***: - The ideal cycle demands the reciprocating elements to move with discontinuous motion which is difficult to achieve. Harmonic motion in reciprocating Stirling cooler can be produced using crankshaft, connecting rod arrangement etc.

c) ***Non Isothermal Compression and Expansion***: - The condition of isothermality can never be achieved, because it requires either infinite rate of heat transfer or engine running at very low speed. In practice engines can nearly run at 25 HZ to 30 HZ and the conditions in the cylinders are closer to adiabatic condition. The departure from isothermal operations causes a noticeable redistribution of working fluid in the machine resulting in decrease in COP.

d) ***Friction Loss***: - The regenerator pressure drop has a direct and immediate effect on refrigeration capacity and should be kept minimum. The pressure drop is a function of square of velocity of fluid. Thus for minimum pressure drop, velocity should be low which can be achieved by large porous matrix of regenerator. Unfortunately, this increases the dead space which deteriorates the refrigerating capacity. The dead space therefore, must be as small as possible. This requires optimization of regenerator.

e) ***Regenerator Contamination***: - Normally a regenerator with effectiveness more than 0.95 is used in Free Piston Stirling Cooler. The regenerator matrix is constructed finely divided material. Unfortunately, this acts as an excellent filter and effectively strains out any contaminants present. Blockage of regenerator passage increases the fluid friction resulting into large pressure drop.

f) ***Regenerator Thermal Saturation***: - Along the length the temperature of matrix decreases progressively (hot end to cold end) although with an ideal regenerator the temperature at any point in matrix is assumed to be constant. This is true when thermal capacity of matrix is infinitely large compared with working fluid. When the heat capacity of gas becomes significant relatively, there is deviation from ideality and the regenerator matrix approaches the state of thermal saturation.

3 Mathematical Formulation

Dead Space Volume

Assume that the hot space, regenerator and cold space dead volumes, in m^3, are respectively V_{SH}, V_{SR} and V_{SC}, then, the total dead volume is;

$$V_S = V_{SH} + V_{SR} + V_{SC} = (k_{SH} + k_{SR} + k_{SC}) \times V_S \qquad (1)$$

where $k_{SH} = V_{SH}/V_S$ is the hot space dead volume ratio, $k_{SR} = V_{SR}/V_S$ is the regenerator dead volume ratio and $k_{SC} = V_{SC}/V_S$ is the cold space dead volume ratio.

Let the total dead volume to total volume ratio be represented as $k_{ST} = V_S/V$, then the total dead volume can be expressed in terms of total volume as;

$$V_S = k_{ST} \times V = k_{ST} \times (V_S + V_P + V_D) \qquad (2)$$

International Journal of Scientific & Engineering Research, Volume 7, Issue 11, November-2016
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where V_D and V_P are the displacer and power piston swept volumes in m³, respectively.

The dead volume is more conveniently expressed in terms of the total swept volume as:

$$V_S = k_{SDP} \times (V_P + V_D) \quad (3)$$

Therefore, the dead volume to the total volume ratio, and the dead volume to the total swept volume ratio are related by:

$$k_{ST} = \frac{k_{SDP}}{1+k_{SDP}} \quad (4)$$

$$k_{SDP} = \frac{k_{ST}}{1-k_{ST}} \quad (5)$$

Regenerator Effectiveness

The regenerator effectiveness, while heating, of an imperfect regenerator is defined as;

$$e = \frac{T_{3\prime}-T_1}{T_3-T_1} \quad (6)$$

The regenerator effectiveness, while heating, of an imperfect regenerator is defined as;

$$e = \frac{T_3-T_{1\prime}}{T_3-T_1} \quad (7)$$

The value of e is 1 for 100% effectiveness or ideal regeneration and e is 0 for 0% effectiveness or no regeneration. The working fluid temperature at the regenerator outlet can be expressed in terms of the regenerator effectiveness as:

$$T_{3'} = T_1 + e \times (T_3 - T_1) \quad (8)$$

$$T_{1'} = T_3 - e \times (T_3 - T_1) \quad (9)$$

For the Stirling engines with large dead volumes, having the correct working fluid temperature for the regenerator is important. The effective temperature of the working fluid contained in the regenerator dead space can be determined using the arithmetic mean;

$$T_R = \frac{\left(T_{1'}+T_{3'}\right)}{2} = \frac{(T_1+T_3)}{2} \quad (10)$$

Equation of State

Assume that the hot space and cold space volumes are, respectively, V_H and V_C and that the working fluid temperatures in the hot space, regenerator, and cold space are, respectively, T_3, T_R and T_1. The equation of state for the isothermal compression process 1-2 with dead volumes V_{SH}, V_{SR} and V_{SC} is

$$p = \frac{m\times R}{\frac{V_H}{T_3}+\frac{V_{SH}}{T_3}+\frac{V_{SR}}{T_R}+\frac{V_{SC}}{T_1}+\frac{V_C}{T_1}} = \frac{m\times R}{\frac{V_H}{T_3}+K+\frac{V_C}{T_1}} \quad (11)$$

$$\text{Where, } K = \frac{V_{SH}}{T_3}+\frac{V_{SR}}{T_R}+\frac{V_{SC}}{T_1}$$

Isothermal Compression Process

In the compression process, the hot-side working fluid is compressed from $V_{C1} = V_D + V_P$ to $V_{C2} = V_D$. The cold-space working fluid swept volume, V_C, is 0 throughout this process.

Then the heat rejected during the isothermal compression process 1-2 is:

$$\begin{aligned} Q_{1-2} = W_{1-2} &= \dot{m} \times R \times T_3 \times \int_{V_{C1}}^{V_{C2}} \frac{dV_C}{(V_C + KT_3)} \\ &= \dot{m} \times R \times T_3 \times \ln\left(\frac{V_{C2}+KT_3}{V_{C1}+KT_3}\right) \\ &= \dot{m} \times R \times T_3 \times \ln\left(\frac{V_D+KT_3}{V_D+V_P+KT_3}\right) \quad (12) \end{aligned}$$

Isochoric Heat Addition Process

Ideally, the constant volume heat addition process is given by;

$$Q_{4-1} = \dot{m} \times C_V \times (T_3 - T_1) \quad (13)$$

where C_V is the specific heat at constant volume in J/kg K, and is assumed to be constant. Without regeneration, this amount of heat is added by an external source and for ideal regeneration this amount of heat is released from an ideal regenerator. The regeneration heat released from an imperfect regenerator during this process is:

$$\begin{aligned} Q_{4-1'} &= \dot{m} \times C_V \times (T_3 - T_{1'}) \\ &= e \times \dot{m} \times C_V \times (T_3 - T_1) \quad (14) \end{aligned}$$

Heat added from an external source during process 1'– 1 is:

$$Q_{1-1'} = (1-e) \times \dot{m} \times C_V \times (T_3 - T_1) \quad (15)$$

Isothermal Expansion Process

In the expansion process, the cold-side working fluid is expanded from $V_{C2} = V_D$ to $V_{C1} = V_D + V_P$. The hot-space working fluid swept volume, V_D, is 0 throughout this process. Then the heat absorbed during the isothermal expansion process 3-4 is:

$$\begin{aligned} Q_{3-4} = W_{3-4} &= \dot{m} \times R \times T_1 \times \int_{V_{C2}}^{V_{C1}} \frac{dV_C}{(V_C + KT_1)} \\ &= \dot{m} \times R \times T_1 \times \ln\left(\frac{V_{C1}+KT_1}{V_{C2}+KT_1}\right) \\ &= \dot{m} \times R \times T_1 \times \ln\left(\frac{V_D+V_P+KT_1}{V_D+KT_1}\right) \quad (16) \end{aligned}$$

Isochoric Heat Rejection Process

Ideally, the constant volume heat rejection process is given by;

$$Q_{2-3} = \dot{m} \times C_V \times (T_3 - T_1) \quad (17)$$

where C_V is the specific heat at constant volume in J/kg K, and is assumed to be constant. Without regeneration, this amount of heat is rejected by an external source and for ideal regeneration this amount of heat is released to an ideal regenerator. The regeneration heat accepted by an imperfect regenerator during this process is:

$$\begin{aligned} Q_{2-3'} &= \dot{m} \times C_V \times (T_{3'} - T_1) \\ &= e \times \dot{m} \times C_V \times (T_3 - T_1) \quad (18) \end{aligned}$$

Heat rejected to an external source during process 3'– 3 is:

$$Q_{3-3'} = (1-e) \times \dot{m} \times C_V \times (T_3 - T_1) \quad (19)$$

Total Heat Addition

For an imperfect regenerator, the heat addition to the working fluid comes during external isochoric heating (over and above heat provided from regenerator) and

International Journal of Scientific & Engineering Research, Volume 7, Issue 11, November-2016
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isothermal expansion. Total heat addition over the entire cycle is given by;

$$Q_L = Q_{11'} + Q_{34}$$

$$= \dot{m} \times C_V \times (1-e)(T_3 - T_1) + \dot{m} \times R \times T_1 \ln\left(\frac{V_D + V_P + kT_1}{V_D + kT_1}\right)$$

$$= \dot{m} \times C_V \times \left[(1-e)(T_3 - T_1) + (\gamma - 1) \times T_1 \ln\left(\frac{V_D+V_P+kT_1}{V_D+kT_1}\right)\right]$$

…….. (20)

Net Work Input

Some of the work done in compression is recovered by the expansion work. However not the entire work done on displacer is transferred to piston due to pressure drops, damping effects and overcoming spring force among others. To take into account loss of expansion work a loss factor K_f is used. Hence the actual or net work that must be supplied is given by;

This **loss factor** is given by;

$$K_f = 0.15 + (T_1 + 20) \times 0.0135 + (30 - T_3) \times 0.025 \qquad (21)$$

$$W = W_{1-2} - K_f \times W_{3-4}$$

$$= \dot{m} \times R \times T_3 \times \ln\left(\frac{V_D + V_P + KT_3}{V_D + KT_3}\right) - K_f \times \dot{m} \times R \times T_1 \times \ln\left(\frac{V_D + V_P + KT_1}{V_D + KT_1}\right)$$

$$= \dot{m} \times R \times \left[T_3 \times \ln\left(\frac{V_D+V_P+KT_3}{V_D+KT_3}\right) - K_f \times T_1 \times \ln\left(\frac{V_D+V_P+KT_1}{V_D+KT_1}\right)\right]$$

….. ……. (22)

Coefficient of Performance

The coefficient of performance is defined as the desired output by the work input. In the case of refrigeration, the desired output is heat removal from the external source i.e. refrigeration space;

$$COP = \frac{Q_{1-1'}}{W}$$

$$= \frac{\dot{m} \times C_V \times \left[(1-e)(T_3 - T_1) + (\gamma - 1) \times T_1 \ln\left(\frac{V_D + V_P + KT_1}{V_D + KT_1}\right)\right]}{\dot{m} \times R \times \left[T_3 \times \ln\left(\frac{V_D + V_P + KT_3}{V_D + KT_3}\right) - K_f \times T_1 \times \ln\left(\frac{V_D + V_P + KT_1}{V_D + KT_1}\right)\right]}$$

$$= \frac{C_V \times \left[(1-e)(T_3-T_1) + (\gamma-1) \times T_1 \ln\left(\frac{V_D+V_P+KT_1}{V_D+KT_1}\right)\right]}{R \times \left[T_3 \times \ln\left(\frac{V_D+V_P+KT_3}{V_D+KT_3}\right) - K_f \times T_1 \times \ln\left(\frac{V_D+V_P+KT_1}{V_D+KT_1}\right)\right]} \qquad (23)$$

4 INITIAL GRAPHICAL ANALYSIS

Using the above formulae, to obtain a deeper understanding of the underlying relationships between variables in the FPSC system, an initial graphical analysis was carried out to plot Coefficient of performance (COP) against changes in input parameters.

Input parameters that were made variable are;

- Dead Space Ratio (k_{ST})
- Regenerator Effectiveness (e)
- Cold side Temperature (T_1)
- Hot side Temperature (T_3)
- Compression Ratio (CR)
- Mass flow rate of refrigerant (m)

4.1 Variation with Hot End Temperature

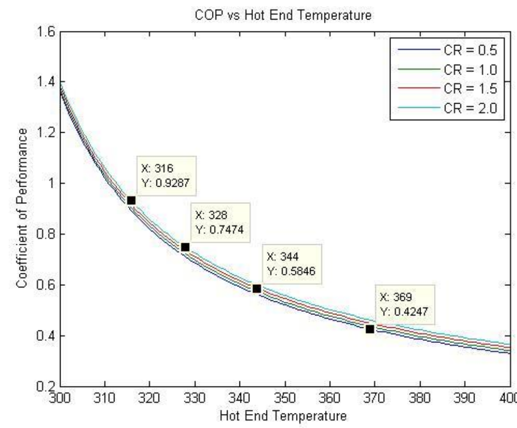

Fig. 3 COP vs Hot End Temperature

4.2 Variation in Mass Flow Rate

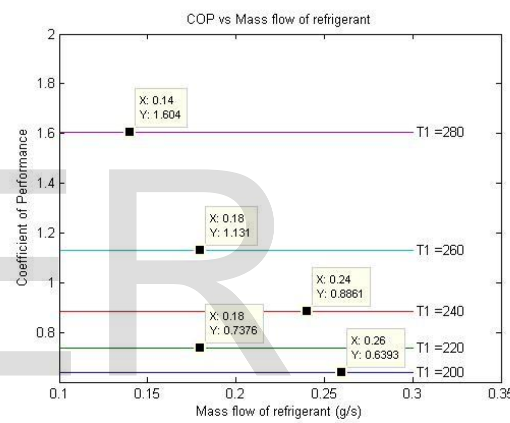

Fig. 4 COP vs Mass flow of refrigerant

4.3 Variation with Cold Head Temperature

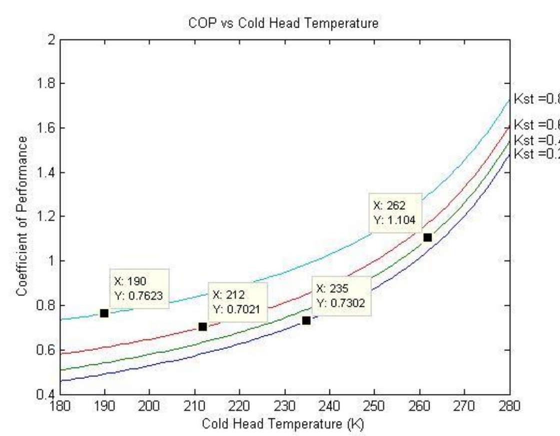

Fig. 5 COP vs cold end temperature

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5. Sample Calculations and Discussion

5.1 Sample Dimensions

The FPSC unit will have the following parameters;

- Hot space temperature T_3 = 308 K
- Cold space temperature T_1 = 255-278 K
- Total effective Volume V = 8500 cc
- Compression Ratio CR = 1.5
- Dead space volume ratio k_{ST} = 0.58
- Hot-space dead volume ratio k_{SH} = 0.2
- Regenerator dead volume ratio k_{SR} = 0.6
- Cold-space dead volume ratio k_{SC} = 0.2

The sample refrigeration space is designed for a small capacity (ideally between 30-40 litres) and having a low heat load.

With regards to the same an insulation thickness of 60mm was decided on the walls except the upper wall to accommodate the cold head of the Free Piston Stirling cooler. Accordingly the top wall uses an insulation thickness of 20mm.

The following are the specifications of the VCC system to be studied in comparison;

- Refrigerant:- R 134a
- Evaporator temperature:- -18°C
- Condensation temperature:- 35°C
- Ambient temperature:- 30°C
- Isentropic efficiency of compressor:- 0.85

5.2 Heat Load Calculations

Heat load calculations for the above mentioned space are carried out and they are given below in Table 1.

Table 1 Heat Load calculations

	Transmission Load (W)	Infiltration Load (W)	Product Load (W)	Total Load (W)
At -18^{0}C				
Steady State	16.542	11.0458	0	27.5878
Unsteady State	16.542	11.0458	3.379	30.9668
At -5^{0}C				
Steady State	12.0618	7.4866	0	19.5484
Unsteady State	12.0618	7.4866	2.464	22.0124
At +5^{0}C				
Steady State	8.6156	4.657	0	13.2726
Unsteady State	8.6156	4.657	1.76	15.0326

5.3 Performance Results

Table 2 Results from evaluations

	Heat Absorbed (W)	Work Input (W)	COP	Loss Factor	Mass Flow Rate of Refrigerant (g/s)
FPSC at -18^{0}C					
Steady State	27.5878	26.0227	1.0601	0.052	0.2469
Unsteady State	30.9668	29.2101	1.0601	0.052	0.2772
FPSC at -5^{0}C					
Steady State	19.5484	15.2226	1.2842	0.2275	0.1726
Unsteady State	22.0124	17.1414	1.2842	0.2275	0.1943
FPSC at 5^{0}C					
Steady State	13.2726	8.5963	1.5440	0.3625	0.1160
Unsteady State	15.0326	9.7362	1.5440	0.3625	0.1314
VCC at -18^{0C}	30.9668	347.176	0.0892		10

The results have also been represented in the form of a bar diagram for reference;

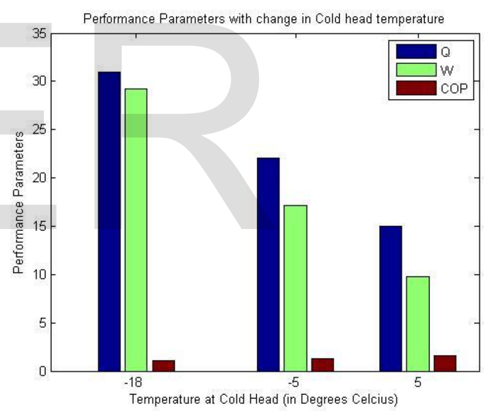

Fig. 6 Results in the form of bar diagram

It is obvious that FPSC refrigeration systems are far superior to their VCC equivalents at low capacities, as can be noted from the huge disparity in their respective Coefficients of Performance (COP).

4 Conclusion

The following conclusions can be drawn from the above analyses;

1) Simplified formulae have been developed for the performance analysis of Free Piston Stirling Coolers (FPSC)
2) A high level of accuracy can be attained with respect to performance parameters such as heat absorbed from refrigeration space, work input required and Coefficient Of Performance (COP)
3) A loss factor has been defined which represents that

amount of expansion work that is used to offset compression work and accounts for other loses as well
4) Free Piston Stirling Cooler (FPSC) has several marked advantages over the traditional Vapour Compression Cycle (VCC) System most significantly the enhanced Coefficient of Performance (COP), environmental friendliness, precise temperature control etc.
5) The system is implementable on a model scale
6) A wide variety of experiments and potential research opportunities can be performed using the FPSC prototype
7) The Free Piston Stirling Cooler is a novel and efficient replacement of the traditional VCC refrigeration system and is the future of refrigeration technology

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